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The Beginning and the End of Urban Population Deconcentration in the United States: New Insights from Application of the Density Distribution Index

Abstract

When did U.S. urban areas begin to experience substantial population deconcentration? Has the deconcentration trend recently ended? These questions have been difficult to answer, in part because many researchers have relied on the density gradient of the negative exponential density-distance function as a valid indicator of population concentration. This article describes an alternative, the Density Distribution Index [DDI], calculated from the density gradient and the central density. Unlike the gradient, the DDI is both mathematically and empirically orthogonal to population size, and thus is unaffected by differing rates of urban growth. This article first calculates DDI scores for metropolitan districts from 1900 to 1970, based on Edmonston's estimates of their density functions (1975). The results indicate that the deconcentration trend took off in earnest at the conclusion of World War II. DDI estimates for SMSAs 1950-1980 (Guterbock, 1990a) show the deconcentration trend continuing strongly through 1980. New density functions are calculated for 119 U.S. urban areas for 1990 to 2015, based on

small area data aggregated into annular rings. These data show that average concentration levels 1990-2015 were virtually unchanged. Two-thirds of the urban areas had higher levels of concentration in 2015 than they had earlier, and this was true of over 90 percent of the largest cities. It can be seen that urban population deconcentration, one of the master trends of the twentieth century, ended for most U.S. urban areas sometime near the start of the twenty-first. The results demonstrate the broader utility of the DDI as a measure of urban concentration.

The Beginning and the End of Urban Population Deconcentration in the United States: New Insights from Application of the Density Distribution Index

Thomas M. Guterbock

This article has three purposes: (1) to introduce to a wider audience the Density Distribution Index, a size-independent measure of urban population concentration based on the negative exponential density function; (2) to show when the trend of rapid population deconcentration actually began for U.S. metropolitan areas; and (3) to show that the deconcentration trend has come to an end for most U.S. urban areas in the last 25-30 years. Published studies of urban density profiles have largely failed to register these important flex points because researchers have not adequately dealt with the ways in which the parameters of the negative exponential density function are related to population size and hence to population growth.

The Problem

Research into urban form has made some progress since Clark (1951) first showed that a negative exponential density-distance function provided a very good fit to the density profiles of urban areas in different time periods and cities around the world. But, as will be seen below, researchers have faced vexing problems in trying to apply Clark's law to gauge urban deconcentration. The problem is not Clark's law, for the negative exponential function continues to fit most urban areas quite well. Rather, the problem lies with how the parameters of that function are usually interpreted. The overall problem was recognized more than sixty years ago in sage remarks by sociologist Otis Dudley Duncan (1959, p. 697):

> This is a field of research with more than ordinary difficulties of conceptualization and measurement. All too often researchers . . . have somewhat naively accepted findings of differential growth rates between central and peripheral portions of urban communities as evidence of a specific process of "suburbanization" or "decentralization," without attempting an operational distinction between these alleged processes and the normal tendency for expansion to occur on the periphery of the community area.

Most urban researchers today do understand the hazards of simply comparing central city growth rates to suburban growth rates; they instead evaluate overall population density profiles in an effort to avoid those hazards (Wang & Zhou, 1999; Broitman & Koomen, 2019; Qiang et al., 2020; Kotharkar & Bahadure, 2020). Many then use *b*, the density gradient, as a measure of each urban area's concentration at a given point in time. In doing so, however, they inadvertently accept the questionable assumption that Duncan pointed to: that any higher growth rates in the periphery than in the centre should be understood as deconcentration.

Theory

In defense of Clark's Law. Clark proposed that the population density of cities is a negative exponential function of distance from the centre, as can be described in the equation

$$D_x = ae^{-bx} \tag{1}$$

where x represents distance from the city's centre, D_x represents the population density at distance x, and e is the base of the system of natural logarithms. If we take the log of both sides, we get a straight line

$$\ln D_x = \ln a - bx \tag{2}$$

with the density gradient b representing the slope of the line, and the log of the central density a representing the line's intercept. Clark aggregated small area data into 1-mile rings around each city centre, and then showed that the logged ring densities generally produced a close fit to the predicted linear pattern.

Researchers have identified several concerns with the negative-exponential density profile. Population density profiles often show a 'density crater' near the centre, where non-residential land uses tend to predominate. The linear fit of logged densities is affected by those deviations. Several researchers have proposed other mathematical functions, such as power, Gaussian or cubic-spline functions that may fit empirical density data more closely (Newling, 1966; Zheng, 1991; see review in Qiang et al., 2020), or other approaches such as the Gini index, the Hoover index, or the ROXY index (Tsai, 2005; Rogerson & Plane, 2012; Kawashima et al., 2014). However, Clark's simple negative-exponential function continues to be the most widely applied, in part because it can be evaluated in the absence of small area data (White, 1977) and also because that density-distance function is supported by the economic models of urban spatial structure developed by Muth (1961) and Mills (1972a).

Aside from the issue of the density crater, there are other concerns over the function's fit. Actual density profiles often vary with direction from the city centre, with higher densities along transportation routes or key amenities (Newling, 1966; Zheng, 1991). Clark's law uses a single centre, but large, modern urban areas are often polycentric. When small area density data for an urban area are graphed in three dimensions, the complex density surface does not conform closely to the smoothly curving density surface implied by Clark's law. Accordingly, when small area data are regressed directly against distance (without aggregating into rings), the fit to a straight line is fairly poor. Small areas (such as census tracts) used for these regression estimates are often smaller and more numerous nearer to the centre than they are nearer to the periphery, requiring the use of weighted regression to make unbiased estimates (Frankena, 1978). This is an element of the larger "modifiable areal unit problem (MAUP)" that addresses how measured density patterns may differ depending on which areal units are chosen (Openshaw, 1983). In a case study of Chicago applying a Monte Carlo simulation to generate alternate spatial units of equal size, Wang et al (2019) go so far as to suggest that the fit of the negative exponential function is an artifact of basing the regression on census-defined units.

With all that said, it remains the case that ~~the same~~ small area data, aggregated into rings as in Clark's original research, generally do fit well to a negative exponential curve. The MAUP issue applies most directly to estimates based on separate small areas; when many small areas are aggregated into annular rings, distortions from their particular shapes or extent should have less influence on the parameters of a regression line fit to the ring data. Guterbock (1990b) analyzed 1-kilometer 'mesh square' data for metropolitan Tokyo in 1975. When the logged population densities of 10,245 grid squares within 65 kilometers of city center were graphed against distance, the linear correlation yielded an R^2 of only .278. When aggregated into 66 1-kilometer rings, the logged ring densities fit closely to a straight line with an R^2 = .958. For Chicago in 2015 (using the new data presented below), if logged densities of 5,940 census block groups (within 45 miles of center) are plotted against distance, the resulting R^2 is only .401; the same block groups aggregated into 46 1-mile rings yield an R^2 = .931. Most recently, Qiang et al. (2020) estimated three different density functions (negative exponential, Gaussian, and inverse power function) for 382 MSA's in the United States in 1990 and 2016, aggregating census block groups into annular "rings" (based on estimated commuting times rather than linear distance). The negative exponential function had the lowest average Root Mean Square Error across all the MSA's, and the authors then use its parameter *b* as their principal measure of concentration. As evidenced by the continued durability of its application, Clark's law is invaluable for the purpose of comparatively describing an urban area's *overall* level of population concentration and how those levels change over time.

Deficiencies of the density gradient as a measure of concentration. The difficulty that our field has had with understanding urban deconcentration processes is because researchers have used the density gradient (*b*) as the primary measure of population concentration. As will be seen below, many have failed to recognize that *b* has complex measurement properties that cause it to be misleading; it is not statistically well-behaved. Moreover, the density gradient cannot be interpreted independently of central density (*a*) because, given population size, they are mathematically interdependent.

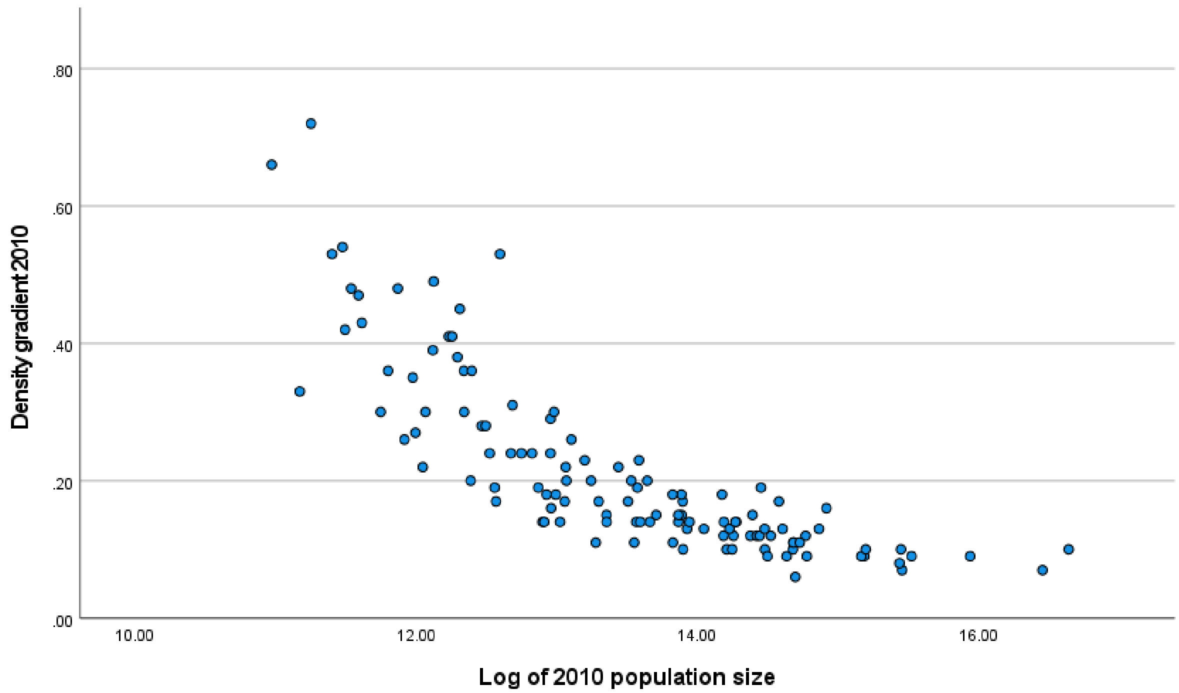


Figure 1. Density gradient by log of 2010 population size for 119 U.S. urban areas.

The density gradient is strongly and negatively correlated with an urban area's population size. This correlation has been noted in many studies (Muth, 1961; Edmonston, 1975; Glickman, 1979; plus others cited in McDonald, 1989). The strongly negative relationship of size to gradient values has been presented as a substantive finding, rather than being recognized as a problematic confound with the measurement of concentration. More rarely, there also have been studies that attempt to account for population size and density function simultaneously (Lemoy & Caruso, 2018). This correlation means that declines in the density gradient's absolute value are strongly associated with the rate of the urban area's population growth. When density gradients for multiple cities are graphed against the log of population size, the resulting scattergram shows a strong negative slope, and much higher variance in values of *b* for smaller cities than for large. Figure 1 shows the relationship from new estimates of density gradients in 2010 for 119 U.S. urban areas (methods described below). The same pattern is visible in Glickman's data for density gradients in Japan (1979; graphed in Guterbock, 1990b), in Rees's data for US SMSA's in 1950 (Rees, 1968, graphed in Berry & Horton, 1970, fig. 9-4b), for Edmonston's density gradient estimates for 1960 Urbanized Areas in the US (1975, graphed in Guterbock, 1982), and most recently for Qiang et al.'s density gradient estimates for US MSA's (2020, fig. 5a).

A related issue with using *b* as the measure of a city's concentration is that when a small city grows, its density gradient declines rapidly, but in a large city a similar rate of growth will produce a lesser decline in *b*. In addition, the higher the initial *b*-value, the easier it is for a given absolute change in *b* to occur. The result is greater volatility of *b*-values for smaller places, as illustrated clearly for U.S. MSAs in Qiang et al. (2020, fig. 5b). These facts make it difficult to interpret changes in the density gradient for urban areas of differing or changing size.

Mills pointed out (1972b, p. 96) that the density gradient remains constant only if all parts of the metro area have equal rates of growth. Higher growth rates in outlying areas will always cause decrease in the density gradient. Thus, if we use the density gradient to measure urban concentration, we implicitly equate stable concentration with equal growth rates at all distances from the centre and with a constant median population distance, since median distance is a function of b alone ($\mathbf{x_m} = 1.678/\mathbf{b}$). If an urban area with a negative-exponential density profile and a given density gradient ***b*** increases in population by ten percent and maintains the same value for the gradient, then population density in every part of the city will have increased by ten percent and the median population distance will remain unchanged despite the increase in size. It is hard to understand why that urban area would not then be considered to be more concentrated than it was before.

In short, if we are to be more successful in comparing degrees of population concentration across urban areas and over time, we need a well-behaved measure of concentration: a measure (unlike *b*) that is unbiased by size and homoscedastic with respect to size, allowing meaningful comparison across urban areas large and small and over time as urban areas grow in population.

The Density Distribution Index [DDI]. The Density Distribution Index or DDI is a size-independent measure of urban population concentration based on the negative exponential density function. It is calculated from an urban area's density gradient (*b*) and central density (*a*) by the following formula:

$$DDI = \ln a + \frac{1}{2}\ln b \tag{3}$$

The justification for this measure begins with Clark's observation (1951, p. 491) that the total population of an urban area with a negative exponential density function is calculated by taking the indefinite integral of the density function, yielding:

$$P_t = ga/b^2 \tag{4}$$

where *g* is the sector size (the arc of settled land surrounding the center) in radians. Taking the log form of the equation, we get

$$\ln P_t = \ln g + \ln a - 2\ln b \tag{5}$$

showing that, for any given sector size, the log of population size is a linear combination of logged *a* and *b*. It follows that (given sector size), if P_t and *a* are known, then *b* is determined; if P_t and *b* are known, then *a* is determined. In other words, there are three quantities (P_t, *a* and *b*) but only two degrees of freedom. Given an urban area's sector size and population size. we can have a relatively flat or steep negative-exponential function. The DDI describes the overall steepness or flatness of that function. As shown in equation 5, the log of total population size is a linear combination of log *a* and log *b*, and DDI is the unique linear function of the same two parameters that is mathematically orthogonal to the log of population size.[1]

The orthogonality of DDI in relation to population size can be illustrated graphically. Graphs developed by Phillip Rees (1968) and published in Berry & Horton (1970, fig. 9-8a) show how any negative-exponential density function can be represented by a single point on a graph that has *ln b* on the x-axis and *ln a* on the y-axis (assuming constant sector size *g*). Reflecting the relationship shown in equation 5 above, a third axis for values of P_t runs obliquely across Rees's graph, increasing from right to left, with population isolines running perpendicular to that axis (see Figure 2).

[1] Mathematical orthogonality of P_t and DDI is demonstrated by the fact that the inner product (dot product) of the coefficients in the two vectors (Equations 3 and 5) is equal to zero: (1*1) + (.5 * -2) = 1 - 1 = 0.

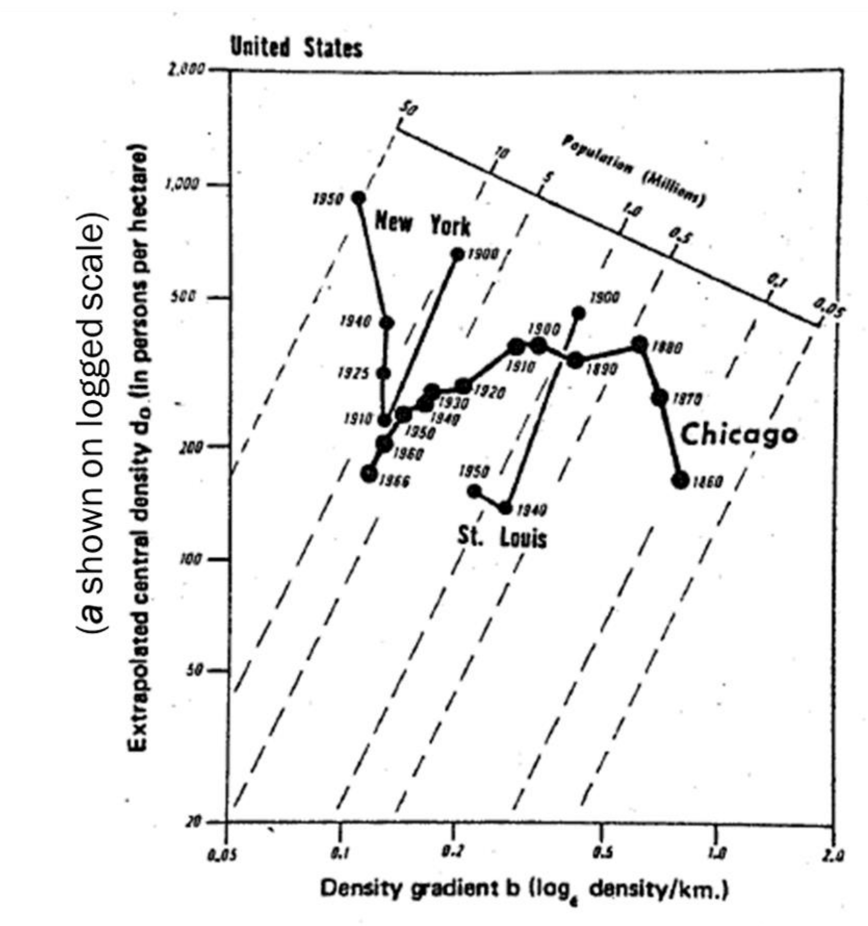


Figure 2. Graph by Phillip Rees showing relationship of central density and density gradient to population size (from Berry & Horton 1970, fig. 9-8a).

Figure 3 illustrates three ways one might assess the degree of population concentration in a city, based on its position in such a graph of its density function. Five hypothetical cities (A through E) are shown in the graph. The conventional practice has been to compare cities in terms of the value of ***b***, the density gradient. By that measure, city E is most concentrated, A the least. In panel b of Figure 3, the cities are compared in terms of their central densities, which would deem city C to be most concentrated and A the least. In practice, researchers sometimes discuss or report central density estimates but do not use them as a primary measure of population concentration (McDonald, 1989, p. 381). In panel c a third axis is introduced, perpendicular to the population axis, representing values of DDI. Accordingly, DDI isolines run perpendicular to the population isolines. When measured by DDI, City A is seen to be far less concentrated than City D, and the rank order of the cities is different. Again, the virtue of this way of measuring population concentration is that it is independent of city size.

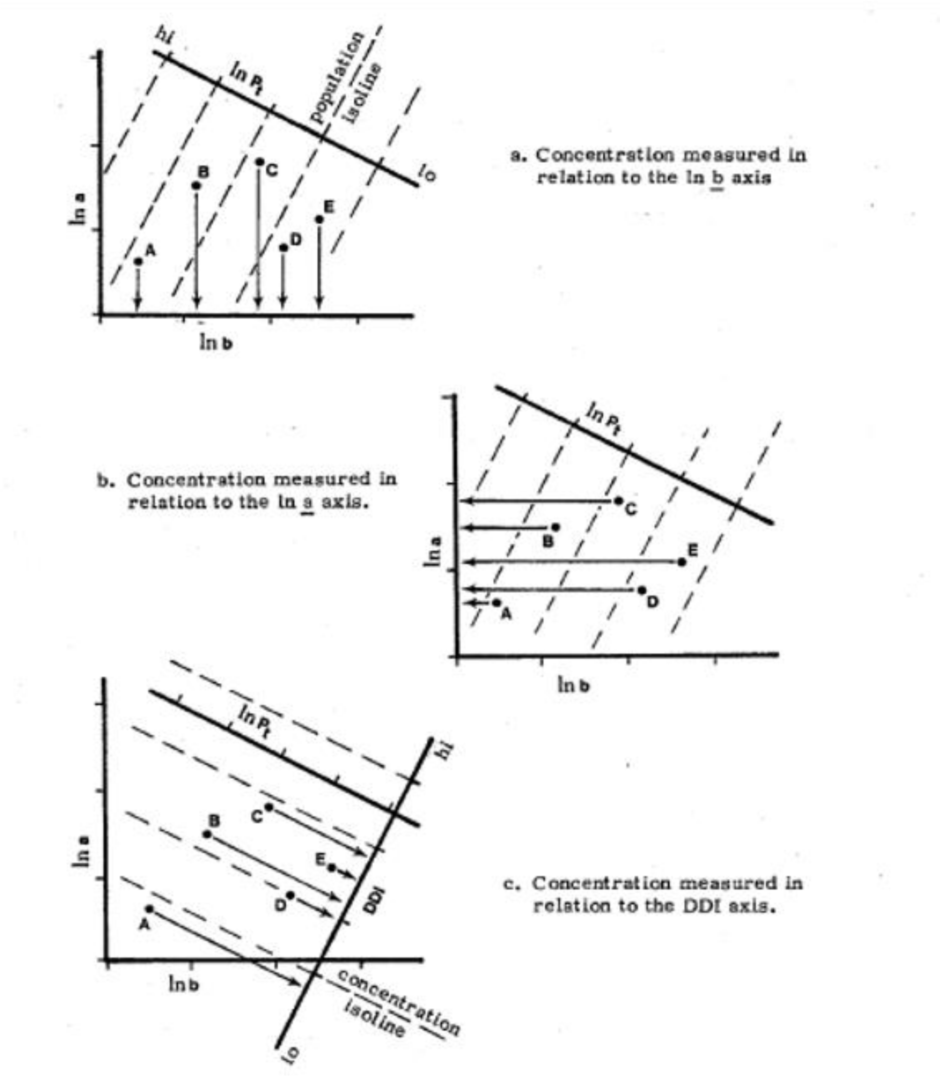


Figure 3. Three ways to measure concentration in a Rees-type graph.

While it is quite true that the form of an urban area's negative exponential density function is defined by *a* and *b* (the main axes in the Rees graph), it is equally valid to think of it as being determined by the two oblique axes in Figure 5(c); that is, by population size and by the degree of concentration of that population, as indicated by DDI. Given the total population size, any change in DDI involves changes in the values of both *a* and *b*.

As was noted above, when *b* is used as a measure of urban concentration we implicitly define constant concentration in a growing urban area as occurring when the rate of increase in population density is the same across all distances from the city centre (Mills, 1972b). In contrast, DDI defines 'normal' growth (growth without change in concentration) as a process in which population density increases near the city centre but increases more rapidly at the periphery. That is, normal growth involves development at the periphery accompanied by somewhat more intense use of land near the centre. Deconcentration is thus defined as a process of population redistribution where peripheral growth occurs without a concomitant increase in densities near the centre. The mathematics of DDI specify what constitutes "concomitant growth." Two cities (city 1 and city 2) (with the same sector size but different population sizes) are equal in DDI if $a_2^2/ a_1^2 = b_1/b_2$. For example, if city 2 has a central density 10 percent higher than city 1, then the two urban areas are equally concentrated only if city 1's density gradient is 21 percent greater than city 2's ($a_2/a_1 = 1.10$, $a_2^2/a_1^2 = 1.21$). The same formula applies to a single city at times 1 and 2. A given city's DDI score would remain unchanged from time 1 to time 2 if growth caused the density gradient to drop by half and the central density to increase by 41 percent ($b_1/b_2 = 2$, $a_2/a_1 = 1.41$, $a_2^2/a_1^2 = 2$).

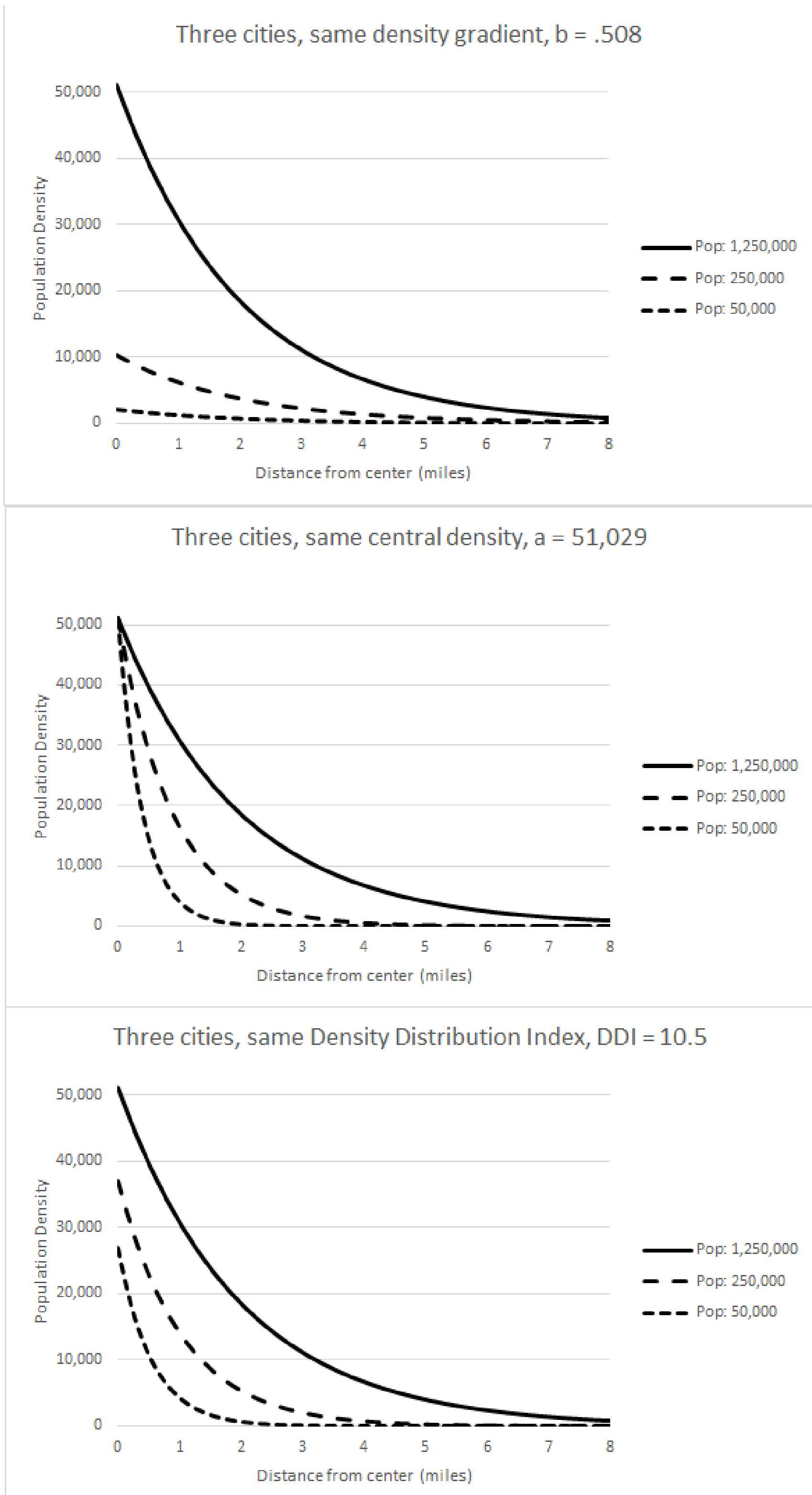


Figure 4. Hypothetical density profiles for cities of different size. Profile of largest city is identical in all three graphs.

Figure 4 shows three hypothetical cities of different population sizes, all with the same level of 'concentration' as defined in three different ways. In Figure 4a, we see cities of population size 50,000, 250,000 and 1,250,000, each having the same density gradient. In each city half the population lives within 3.3 miles (median distance) of the center. In the smallest, 25,000 people live within 3.3 miles; in the largest, 625,000 live within that distance. Most observers would agree that the smallest of these three cities has the least concentrated population.   In Figure 4b, we see three cities with these same population sizes and all having the same central density. Here the problem is the opposite; we would have to consider the density profile for the smallest city, with a central density of over 50,000 persons per square mile, as the most concentrated, while the same central density seems only moderately high in the largest city. In Figure 4c, we see that smaller cities with the same DDI as the larger cities seem similarly compact in relation to their size. In short, density profiles for large and small cities with the same DDI score are congruent in form; they actually look alike.

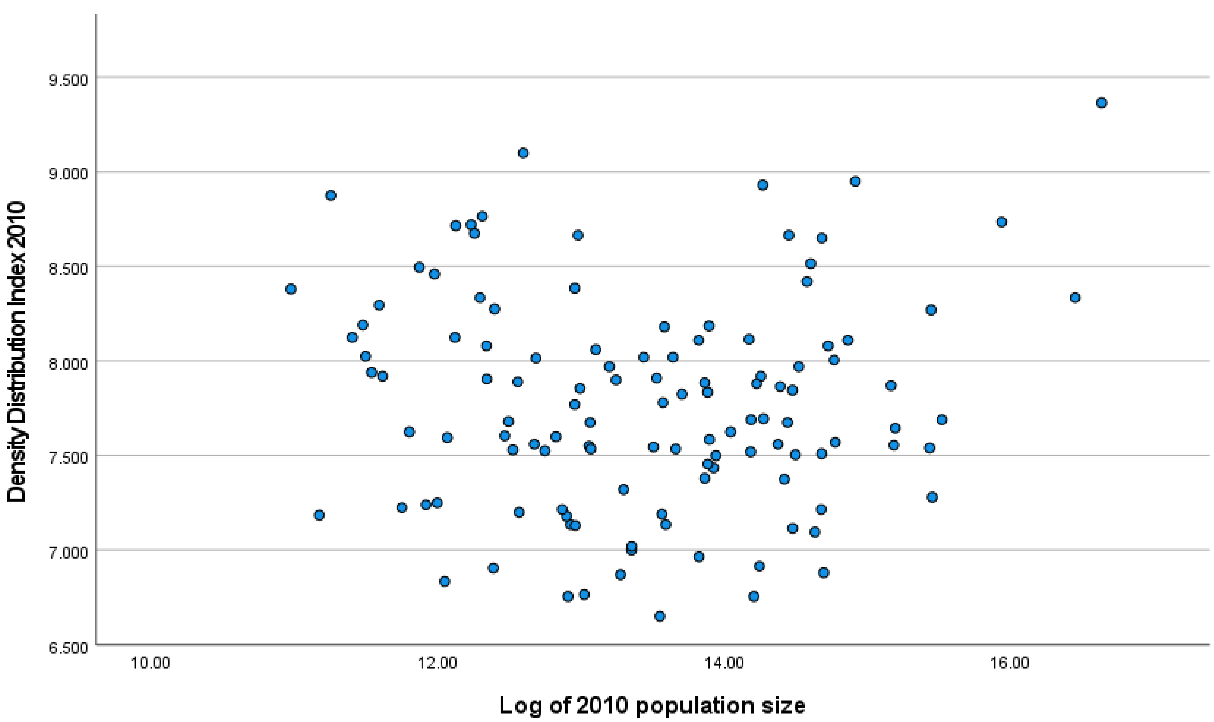


Figure 5. Density Distribution Index by log of 2010 population size for 119 U.S. urban areas.

As was seen in figure 1, the density gradient is negatively correlated with population size and is heteroscedastic in relation to size. Figure 5 shows 2010 DDI scores in relation to logged population for the same 119 U.S. urban areas. There is virtually no correlation between DDI and population size, and the scatter of points is fairly uniform across the range of population sizes. This scatterplot demonstrates that DDI is not only mathematically orthogonal to urban size but empirically independent of it as well. DDI is, by design, a size-independent measure of urban population concentration.

In summary, the two dimensions of the negative-exponential density function, *a* and *b*, can be re-parameterized into the two independent dimensions of *ln Pt* (logged population) and DDI (concentration). DDI incorporates information from both *a* and *b*. DDI scores are mathematically orthogonal and empirically uncorrelated with population size, and DDI scores are satisfactorily homoscedastic with respect to size. Urban areas with the same DDI score are congruent in the form of their raw-form density profiles. It is plausible to treat DDI as an interval-level measure[2] of concentration, allowing us to compare the magnitude of changes in concentration for urban areas of varying size or measure changes in concentration independently of growth. But the case for the DDI as a concentration measure ultimately rests on its construct validity—its utility in identifying theoretically expected patterns (Messick, 1989) in urban population data. That will be demonstrated in the analyses below.

Data and methods.

The analyses use three sets of estimates of negative-exponential functions for U.S. urban areas[3].

[2] Interval level measures allow meaningful comparisons of differences between values, but not ratios. Changing units of measure of distance (miles vs. kilometers) or area (square miles vs. hectares) merely adds or subtracts a constant to the DDI score, leaving intervals and differences unchanged.

[3] "Urban area" is used here as a generic term for any urban agglomeration, with "city" used as a synonym.

1. Edmonston data. Using a method of two-point estimation[4] that takes into account different urban shapes, Edmonston (1975) estimated negative-exponential density functions for over 100 U.S. metropolitan areas from 1900 through 1970, based on decennial census data for central cities and metropolitan districts (1900 to 1940) and for central cities and Census-defined Urbanized Areas (1950 to 1970).

2. SMSA data. Edmonston and Guterbock built upon Edmonston's data set and on Guterbock's methods (Guterbock, 1976). Using a novel but laborious two-point estimation method that more closely controlled for differing urban shapes, Edmonston & Guterbock (1984) calculated negative-exponential density gradients based on central city and SMSA data for a subsample of urban areas from Edmonston's data set, holding city and SMSA boundaries constant within each decade, 1950 to 1980. The subsample was stratified by size and Census region, drawing all of the largest SMSAs and then equal numbers from each of five smaller size classes.

3. New data. In collaboration with urban planner Luke Juday, the author newly calculated negative-exponential density functions for 119 U.S. urban areas, based on decennial census data for 1990, 2000, and 2010, and on American Community Survey five-year estimates for 2013-17, which can be taken as estimates of the 2015 population distribution. The sample of cities includes 100 urban areas from the Guterbock/Edmonston data set (those for which there were valid 1980 density gradient estimates based on two-point estimation from central city and SMSA data), as well as 19 additional cities that were part of Juday's sample (2015). The density gradients are calculated using Clark's original method: aggregating small area data into concentric rings, and then fitting a line by ordinary least squares regression to the log of population density (persons per square mile) vs. the distance from the centre (in

[4] Technically, the term *two-point estimation* is really a misnomer, as these estimates are based on two large integrals encompassing almost the entire metropolitan population, rather than a sample of two points or small areas.

miles). U.S. Census block groups were used as the small areas. This method is unaffected by city boundaries or metropolitan area definitions and is sensitive to population changes in the central business district, in contrast to the estimates of Qiang et al. (2020), which excluded areas in the "density crater." City centres were identified by analysis of relevant map features (as described in Juday, 2015), and in some cases by using the geocoordinates for the location marker for a city as shown in Google Maps. Each block group was assigned to the city centre to which it was closest and was counted in only one urban area. The initial distance for the outermost ring was generally set where the moving average of three successive concentric rings fell below 150 persons per square mile. In some cases the choice of outermost ring had to be adjusted based on inspection of the log-density/distance scattergram.[5] The linear fit of the log-density/distance regression lines was generally good, with regression coefficients ranging from -.99 to -.65. Goodness of fit (as measured by r^2) was uncorrelated with urban area population but did correlate significantly with the value of the density gradient (r = .207, p =.02), with a weaker and notably more variable fit in urban areas having flatter density gradients. This pattern was controlled by simply excluding the few candidate urban areas with regression coefficients below .65. Of the 119 included urban areas, 94 percent had regression coefficients of -.80 or better. The included block groups comprised a combined population of over 169 million people in 2010, 54.7 percent of the total U.S. population.

Results

When did U.S cities start to deconcentrate? We can illustrate the utility of the DDI by looking at a simple descriptive question that has not heretofore had a definitive answer: When did the suburbanization trend start for cities in the United States?

[5] The designated center centre and the outermost ring distance were kept constant for each urban area across the four time points..

Early inquiries into this question looked at differential rates of growth between central cities and their suburbs. Hawley found that “population deconcentration in the relative sense—outlying areas growing more rapidly than central cities—became general after 1920” (1981, pp. 163-164). Schnore’s analysis agreed, concluding that “it appears that decentralization occurred with a rush following World War I,” but noting that New York City had ring growth exceeding central city growth as early as the 1850’s (1957, p. 103). Kasarda & Redfearn (1975) found that, controlling for annexation, “relative decentralization” (defined as a positive differential in growth rates for ring versus central city) has occurred for the average city in every region of the U.S. since 1900. Jackson (1975) pushes the beginning date even further back, showing that Philadelphia showed increasing proportions of its population living outside the central city as early as the mid-nineteenth century. These early studies all relied on contrasting central city growth with growth in surrounding suburbs, and all defined deconcentration as occurring when peripheral growth exceeds growth near the center (see Wang & Zhou for a similar definition in a study fitting alternative density functions to density data from Beijing [1999, p. 283]).

Mills (1972a) addressed the timing question by estimating negative-exponential density gradients for four U.S. cities from 1880 to 1963, using two-point estimates based on central cities and SMSAs (or their equivalents). Based on changes in the density gradients and median population distances, he found “surprisingly little variability from one period to another in the rate at which the metropolitan areas decentralized. There is no evidence at all that the rate of decentralization is more rapid in recent years than in earlier years” (1972a, p. 49). Summarizing these results in a chapter of his widely adopted 1972 urban economics textbook, Mills wrote:

> Generally, suburbanization proceeded most rapidly during prosperous periods, and slowed during depressions. The findings presented in this chapter can be summarized by the statement that suburbanization has been a massive and pervasive characteristic of metropolitan areas in the United States as far back as the record has been examined (1972b, p. 101).

Using the density gradient as the measure of concentration, Edmonston (1975) divided urban areas into age cohorts (based on the year at which the city reached a population size of 50,000) and conducted analyses for 1900-1970 based on metropolitan size, region, age cohort and time period. Grouping Edmonston's cities into seven age cohorts, Figure 6 shows how the average density gradient for each cohort of U.S. urban areas changed over the successive decades.

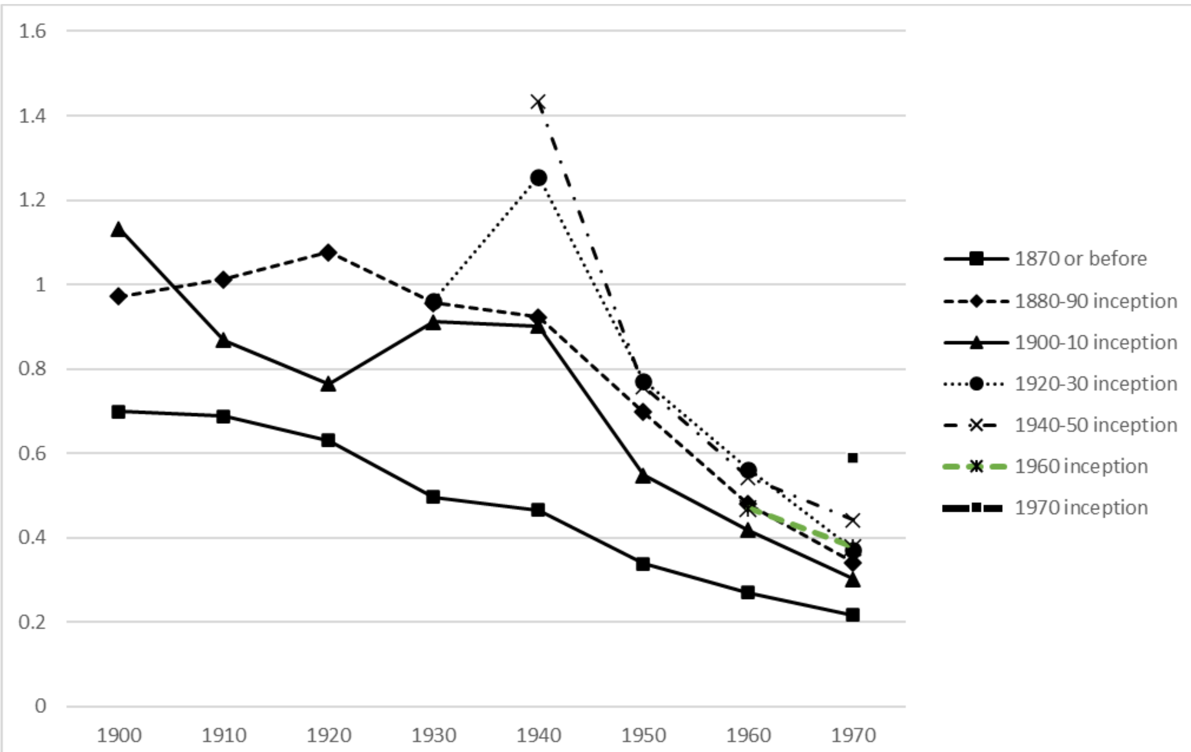


Figure 6. Average density gradient, 1900-1970, for seven age cohorts of U.S. cities

The generally smooth descent of the trend line for the oldest cohort replicates Mills' findings (1972a) for the four old cities he studied. The fact that younger cohorts show higher gradients at each point in time echoes the idea of 'metropolitan maturity' suggested by Schnore (1957); cities are 'born' with high density gradients but their gradients flatten as they age. The trend lines, taken together, do not suggest any acceleration of the deconcentration process. Taking all cities together, the graph suggests that deconcentration proceeded rather steadily, with a pause in the depression decade of the 1930s that was then made up for by a plunge in concentration in the 1940s.

These interpretations assume that the density gradient is, in itself, an adequate measure of urban concentration. As has already been discussed, however, larger cities have flatter gradients, and rapidly growing cities therefore have greater decreases in their density gradients over time. Moreover, steep gradients change more readily than flatter gradients. The complex pattern in Figure 6 is actually reflective of the confounding effects of city size and growth rates on the density gradient.

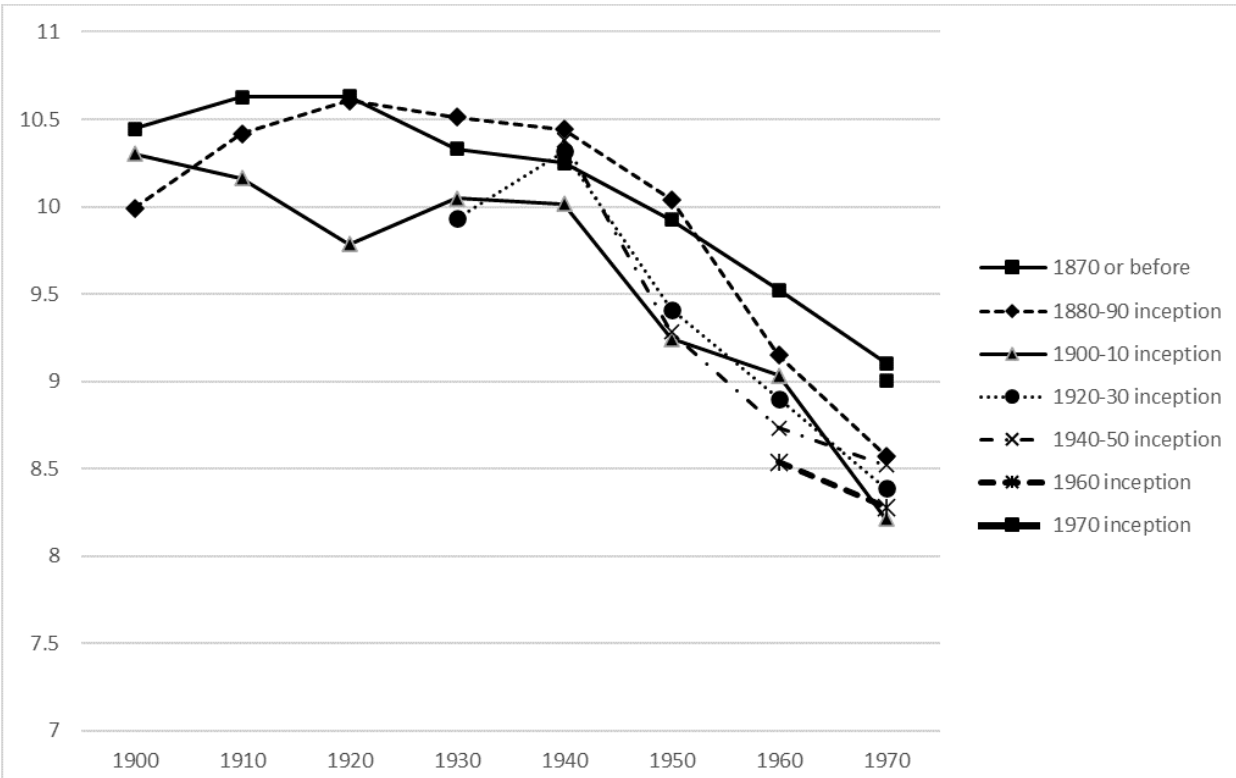


Figure 7. Average value of the Density Distribution Index, by date of metropolitan inception

Figure 7 displays data for exactly the same groups of cities, but uses the DDI as the measure of concentration, calculated from Edmonston's original density function estimates (1975). It contrasts sharply with the complex patterns seen in Figure 6; the apparent cohort differences virtually disappear. As new cities reach the threshold size of 50,000, they display DDI scores similar to the contemporaneous DDI scores of older cohorts. The higher density gradients of the younger cities (seen in Figure 6) are merely a product of their smaller population sizes, not a product of city age (or lack of 'metropolitan maturity') itself. There are some interesting differences among the cohorts, but these are small differences that are overshadowed by the clear pattern of change over time. (For additional details on this analysis, see Guterbock, 1982.) From 1900 to 1940, U.S. metropolitan districts—young and old, large and small—maintained a fairly steady level of population concentration whether growing rapidly or not, with some deconcentration occurring in the decade of the 1920s. Then, after 1940, there is a clear inflection point as urban areas start rapid and steady deconcentration through 1970. In Figure 7, with the confounding effects of size and growth removed, we can clearly see that the rapid suburbanization trend for U.S cities began with the end of World War II, with no evidence of deceleration in that trend up to 1970 at least. This historic shift in urban form was obscured in Figure 6 by the confounding effects of size and varying growth rates. In addition, the older cohorts in Figure 7 generally have higher levels of concentration (as measured by DDI) than newer cohorts, as expected theoretically based on their pre-automobile initial development; in Figure 6 older cohorts show lower density gradients (due to their larger average size). By using a size-independent measure of concentration, the DDI, we can clearly identify when U.S. cities actually started their "massive and pervasive" trend of population deconcentration.

Has urban deconcentration come to an end in the U.S.? The deconcentration trend continued beyond 1970. Table 1 and Figure 8 show how the estimated DDI scores for U.S urban areas changed within each decade from 1900 to 2015, beginning with Edmonston's (1975) estimates for 1900 to 1970. Average concentration was still rising from 1900 to 1920, then leveled off and declined noticeably after 1920, validating the observations of Schnore (1957) and Hawley (1981). However, as already depicted visually in Figure 7, rapid deconcentration started in the 1940s with the post-war boom in housing. Table 1 and Figure 8 continue the time series with SMSA estimates for 1950 to 1980 from the work of Guterbock and Edmonston. These data suggest that the rapid pace of deconcentration continued steadily through 1980 (see also Edmonston & Guterbock, 1984). Now, four decades later, we must wonder: Is suburbanization slowing down at last?

__Table 1. Change in average DDI scores, 1900-2015__

				Decreased DDI		**Increased DDI**	
Year	**n**	**Mean DDI**	**Inter-decade change**	n	%	n	%
1900[a]	39	10.14	0.363	3	7.7	36	92.3
1910	39	10.50					
1910	40	10.51	0.027	15	37.5	25	62.5
1920	40	10.54					
1920	50	10.44	-0.125	37	74.0	13	26.0
1930	50	10.32					
1930	74	10.28	-0.088	61	82.4	13	17.6
1940	74	10.19					
1940	88	10.30	-0.725	84	95.5	4	4.5
1950	88	9.58					
1950	103	9.56	-0.420	80	77.7	23	22.3
1960	103	9.14					
1960	113	9.06	-0.590	101	89.4	12	10.6
1970	113	8.47					
1950[b]	83	10.16	-0.369	72	86.7	11	13.3
1960	83	9.79					
1960	94	9.53	-0.174	77	81.9	17	18.1
1970	94	9.36					
1970	101	9.01	-0.339	93	92.1	8	7.9
1980	101	8.67					
1990[c]	119	7.85	--				
2000	119	7.82	-0.032	70	58.8	49	41.2
2010	119	7.78	-0.044	74	62.2	45	37.8
2015	119	7.79	0.009	57	47.9	62	52.1

[a]1900-1950 DDI calculated from Edmonston's two-point estimates for metro districts (1975)

[B]1950-1980 DDI calculated from two-point estimates for SMSA's, boundaries constant in each decade [REF DELETED]

[c]1990-2015 estimates from current analysis of block-group data

There are reasons to think that the deconcentration trend must have an endpoint. The consensus in urban economics and in urban studies more generally is that urban density profiles are primarily determined by economic, social and technological conditions: the means and costs of transportation and communication, changing household size and incomes, changing tastes in housing and lifestyles, etc. (Mieszkowski & Mills, 1993). (The effect of urban planning and government policies on the overall density profile is less clear; see Bontje 2001.) Cities that were built under earlier conditions, conducive to high levels of concentration, are expected to spread out as warranted by newer conditions, such as the rise of the automobile and the Internet. But eventually, urban form should reach an equilibrium with these newer conditions, while some conditions may change in a direction more conducive to concentration of urban populations. Recent studies of European cities, using a variety of methods, suggest that a process of "reurbanization" may be underway (Bromley et al., 2005; Tallon, 2013; Kroll & Kabisch, 2012; Broitman & Koomen, 2019).

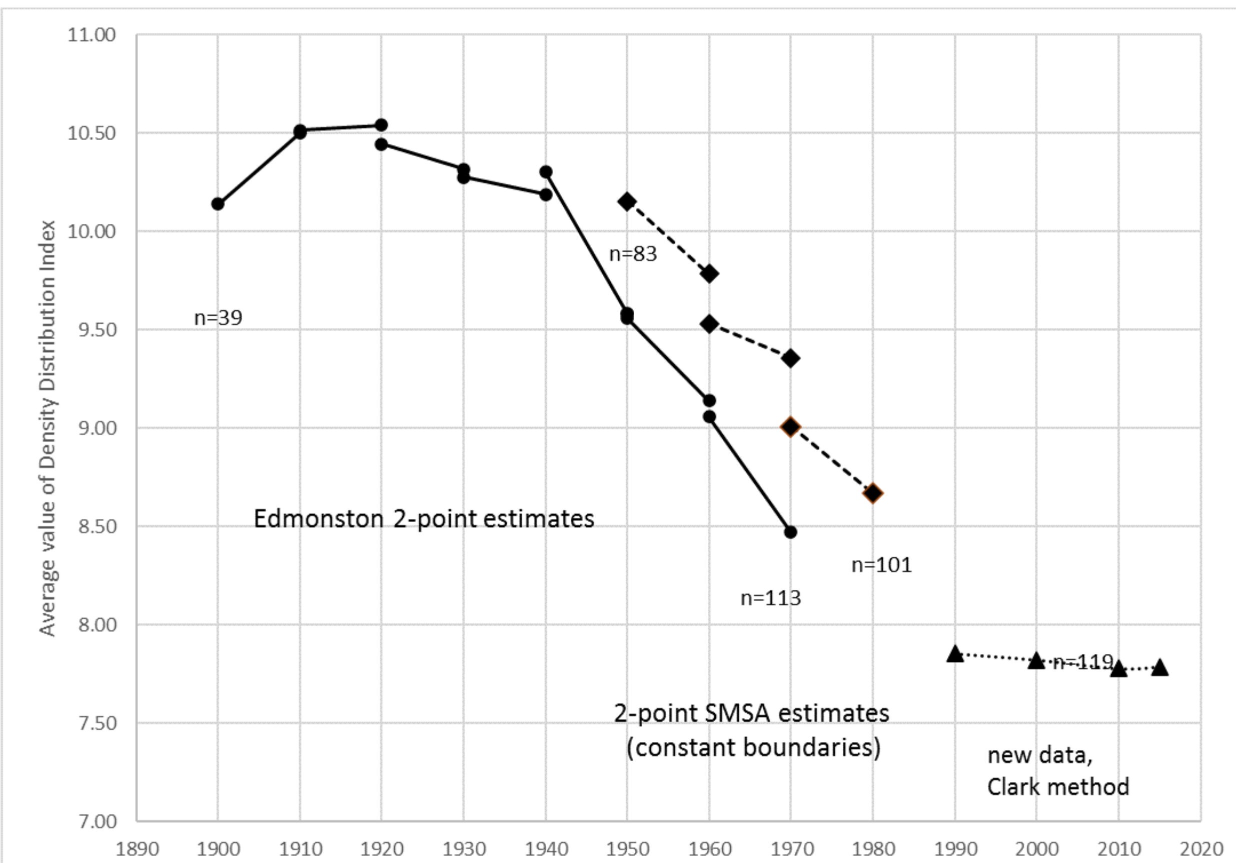


Figure 8. DDI Estimates 1900-2015, growing number of cases

Reports of urban revival in the U.S. are perennial, going back to the 1970s (e.g., Lipton, 1977). The oil crisis of that decade led to incorrect predictions that cities would become more compact (Robson & Schiffman, 1979). But the changes in downtown areas of some U.S. cities in recent decades have been more dramatic and larger in scale. These changes are evident in the graphs of 66 urban-area population density profiles published online by the Demographics Research Group at the University of Virginia Weldon Cooper Center for Public Service (Juday, 2015). The density-distance graphs, which Juday created by aggregating Census block-groups into concentric rings, show dramatic increases from 1990 to 2013 in densities near the city centre in many cities. U.S. Census population estimates for 2019 showed that 24 out of the 25 largest U.S. central cities experienced population increases compared to 2010 (U.S. Bureau of the Census 2019). While this pattern represents a dramatic contrast to the population losses many of these cities experienced after 1970, we cannot conclude from these data alone that the larger metropolitan areas were becoming more concentrated after 2010. Suburban and exurban areas were growing too, usually more rapidly than the central cities.

In recent work that suggests continued population deconcentration in U.S. cities, Qiang et al. (2020) calculated urban density gradients for 382 U.S. MSAs. Comparing 1990 to 2012-16, they concluded that 70 percent of the MSAs had a "decentralization trend" over that period. They based this conclusion on the fact that 70.7 percent showed a decline in density gradient, *b*. As has been argued above, however, *b* is sensitive to urban growth as well as to concentration itself; some of the decreases in *b* may merely reflect continued population growth. As that work illustrates, despite obvious and well-known revitalization and even absolute population growth in many large central cities in the U.S., relying on the density gradient has made it difficult to understand or measure the overall trend. We all know of U.S. cities that are attracting new households to their centers (on New York, see Barron 2018; on Washington, see Rowlands 2019); we can see the trees but have not had a good lens through which we might see the forest.

By using the Density Distribution Index, we can more clearly identify whether U.S. urban areas have continued the deconcentrating trend that started in the middle of the last century. Table 1 and Figure 8 include the average DDI values for cities in the newly created dataset for each decade from 1990 to 2010, and for 2015. The averages are remarkably stable over this 25-year period; rounded to one decimal the average DDI score for the 119 cities remains constant at around 7.8 throughout the period. The inter-period changes in average DDI from 1990 to 2015 are miniscule. This is a sharp contrast to the average DDI values for the period 1950-1980 from the two-point estimates in the Guterbock/Edmonston dataset. It is fair to say that, based on the DDI score as our measure of urban concentration, U.S. cities, on average, did not substantially deconcentrate after 1990.[6]

In contrast, the estimated density gradients for this set of cities show a pattern of decline even stronger than that reported by Qiang et al. (2020). From 1990 to 2015, 114 of these cities (96%) show a decrease in the density gradient. However, we have seen that *b* is sensitive to population growth, and in the same period the total population contained in the 119 urban areas increased by 31.3 percent.

The apparent stability in average DDI from 1990 to 2015 reflects the fact that some cities were becoming more concentrated while others were still deconcentrating. Table 1 shows the number of urban areas in each time period that experienced a decrease or an increase in DDI score. In the period of rapid suburbanization from 1940 through 1980, the vast majority of SMSAs were deconcentrating in each period, including over 90 percent of SMSA's in the decade of the 70s. In contrast, the percentage of urban areas showing *increased* population concentration after 1990 is substantially higher, with over half of the 119 sampled cities showing an increase in DDI in the five-year period from 2010 to 2015.

[6] The data in Table 1 cannot properly be used to calculate intra-decade change from 1980 to 1990, as the datasets used in the two tables differ in the estimation method, the sample of included cities, and the geography used in each urban area.

Some urban areas reversed their suburbanization trend earlier than others. We can consider any urban area that shows a higher DDI score in 2015 than in some prior year 1990-2000 as having experienced a bounce-back from the deconcentration trend. Two-thirds of the urban areas in our current sample (80 out of 119) have a 2015 DDI score that is higher than the lowest DDI score recorded from 1990 to 2010. Breaking the sample into groups by population size, 94 percent of the 35 sampled urban areas with 2010 populations of 1,500,000 or more show a positive 'bounce' in DDI compared to their lowest score in the preceding two decades. For the 23 urban areas between 750,000 and 1,500,000 in population size, about 70 percent experienced a bounce in DDI. For 60 cities smaller than 750,000, only 44 percent had reversed their trend of deconcentration by 2015.

The time of reversal (if any) of the deconcentration trend also varied across the 119 urban areas. About 25 percent had their lowest DDI score as early as 1990, another 20 percent were least deconcentrated in 2000, 22 percent were lowest in 2010, and one-third were lowest in 2015, meaning that they had not yet started to become more concentrated. Larger cities bounced back from the deconcentration trend earlier than smaller cities, with 58 percent of the urban areas over 1,500,000 in 2010 population having their lowest point of concentration in 1990 or 2000, compared to 28 percent of cities smaller than

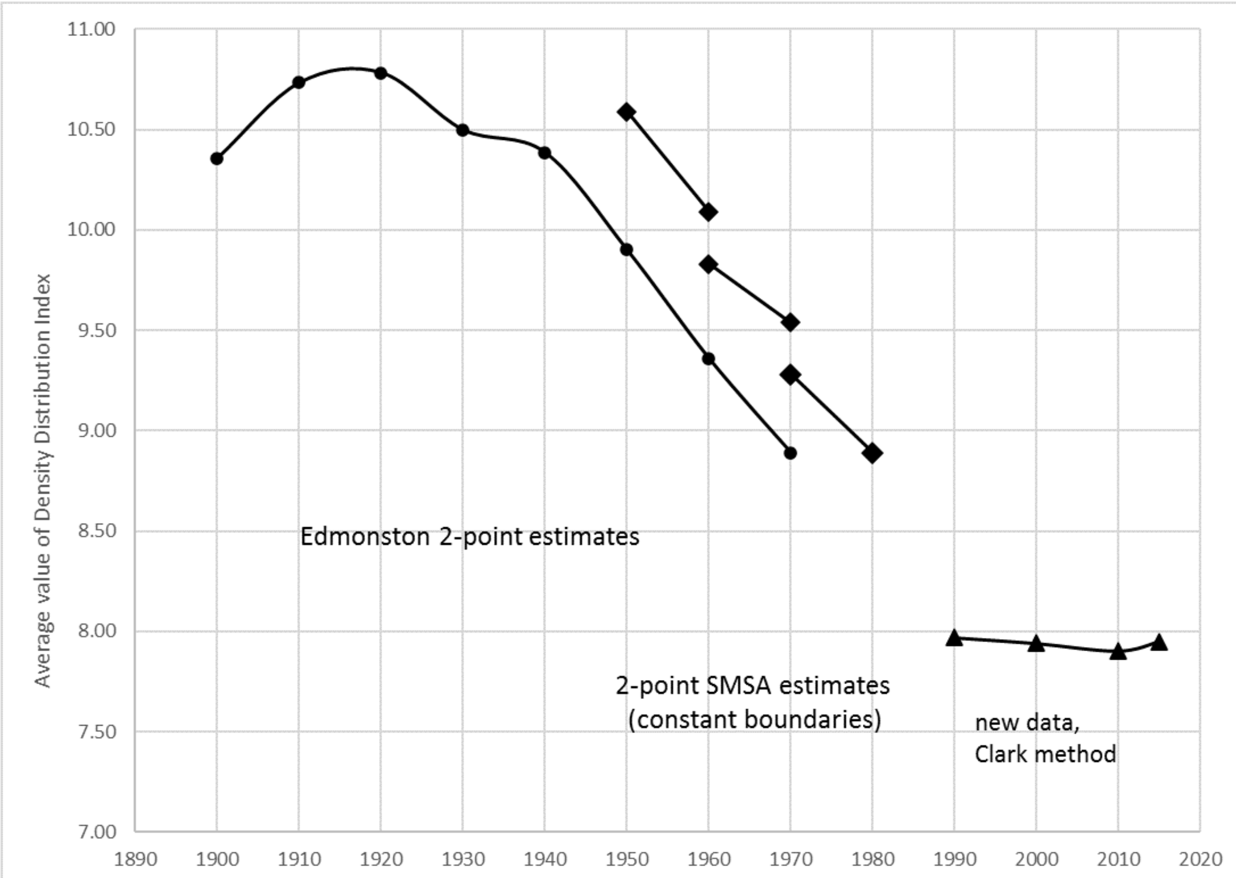


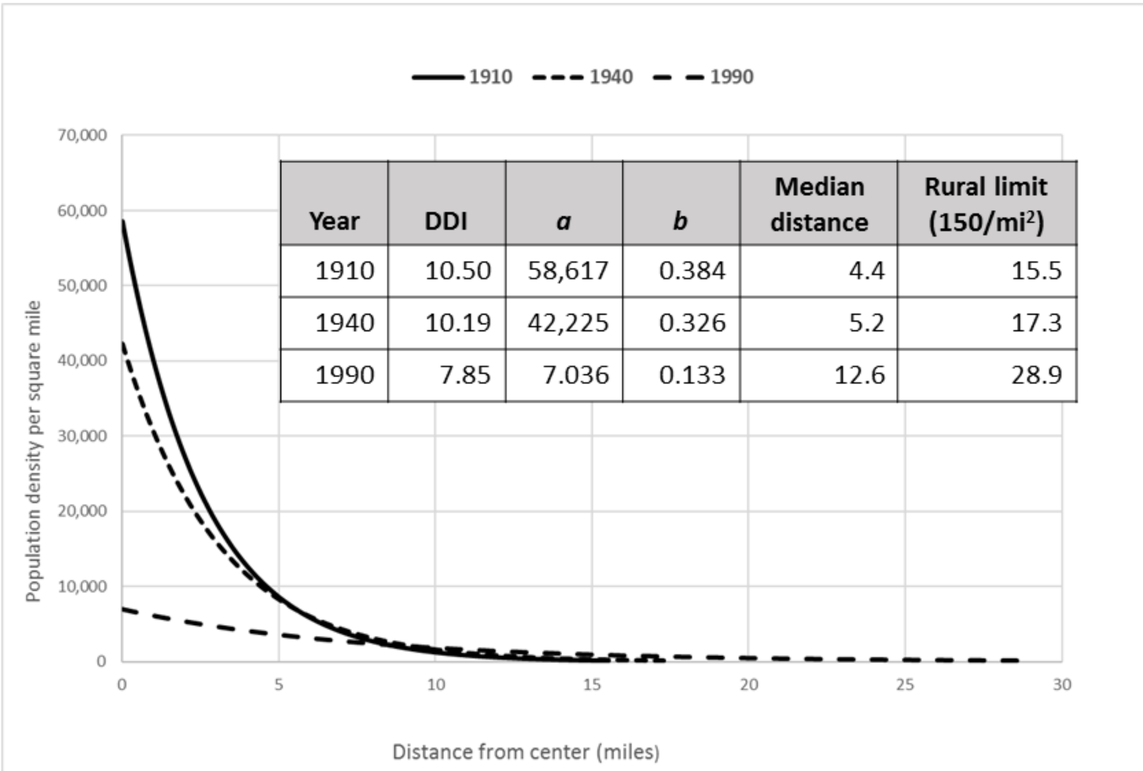
Figure 9. DDI Estimates 1900-2015: Fixed set of 28 urban areas.

1,500,000. Further analysis of why some cities bounced back from their lowest concentration levels earlier than others is beyond the scope of this article.

In Figure 8, the sample of cities is held constant within each decade but grows over time. Figure 9 shows average DDI scores for a constant subset of 28 cities for which estimates could be made for every time point.[7] The pattern of change for the constant cohort mirrors the pattern seen in Figure 8. Notably, the average DDI for the cohort is consistently higher than the average for the full sample, because the

[7] The cities are: Atlanta, Baltimore, Birmingham, Boston, Bridgeport, Buffalo, Chicago, Cincinnati, Cleveland, Columbus, Dayton, Denver, Detroit, Grand Rapids, Louisville, Milwaukee, Minneapolis, New York, Philadelphia, Pittsburgh, Portland, Richmond, Rochester, Spokane, St Louis, Syracuse, Toledo, and Washington.

constant cohort consists of cities with inception dates before 1910; older cities, as expected, are more concentrated than newer cities (compare McDonald, 1989, p. 380).



Year	DDI	*a*	*b*	Median distance	Rural limit (150/mi^2)
1910	10.50	58,617	0.384	4.4	15.5
1940	10.19	42,225	0.326	5.2	17.3
1990	7.85	7.036	0.133	12.6	28.9

Figure 10. Density profile of hypothetical cities: 1910, 1940, 1990 average DDI (population 2,500,000).

Figure 10 illustrates the density profiles for a hypothetical, circular urban area with a population of 2,500,000 if its DDI score equaled the average for cities in 1910 (the year of peak concentration), 1940 and 1990. These density profiles illustrate that the deconcentration that occurred between 1910 and 1940 was substantial, but fairly limited compared to the drastic transformation in population concentration that occurred between 1940 and 1990. If a density of 150 persons per square mile is taken to represent the limit of settlement at urban densities, then the urbanized land area defined by these profiles would have expanded from 757 square miles in 1900 to 938 in 1940 and then to 2,620 square miles in 1990.

Discussion

Urban researchers who analyze the overall population concentration of urban areas often calculate negative-exponential density-distance functions for each urban area, taking the density gradient (*b*) as the parameter that represents an urban area's degree of population concentration. This article has highlighted the fact that *b* is highly dependent on an urban area's population size, and that therefore changes in the value of *b* are dependent on the amount of population growth. Because the density gradient is confounded by population size and population growth (a circumstance of which many researchers seem to be insufficiently aware), researchers have faced vexing difficulties in describing and explaining between-city differences in concentration levels and over-time changes in those levels.

This article has described the Density Distribution Index, a size-independent measure of urban concentration. The DDI is both mathematically and empirically orthogonal to the population size of an urban area, and therefore allows valid comparison of large and small cities, or cities that are growing to others that may be stable or declining in size. The DDI implies a specific definition of stable or similar concentration levels, one that requires a concomitant increase in density near the centre when there is increased density at the periphery. When population grows, this definition allows for greater rates of change in density at the urban periphery than near the centre. (In contrast, *b* remains constant only when the rate of growth is identical at all distances from the city centre.)

Re-analyzing data collected by Edmonston (1975) for U.S. urban areas from 1900 to 1970, I have shown that density-gradient (*b*) levels and changes vary widely across age cohorts of urban areas, due primarily to the fact that the cohorts vary greatly in average population size (older areas are larger) and in rates of growth. When the same data are examined using the DDI instead of the density gradient, the concentration levels of all city-age cohorts are seen to be fairly similar at each time point. Urban deconcentration started around 1920, took off after World War II and continued unabated through the decade of the 1970's. Guterbock's later work with Edmonston showed that trend continuing up through 1980. Thanks to the DDI, we can confidently answer the question of when rapid suburbanization began in U.S. urban areas: around 1945.

Looking at DDI scores from density functions for 119 U.S. urban areas from 1990 through 2015, newly calculated using Clark's (1951) original method of concentric rings, we have seen that some urban areas started to become *more* concentrated in the 1990s, some in the 2000s, and some after 2010. About two-thirds of the urban areas in this sample experienced an upward bounce in population concentration sometime in the 25-year period 1990-2015. This was true of over 90 percent of the larger cities in the sample (those over 1,500,000 in 2010 population size). The larger cities were more likely to experience the concentration turnaround in 1990 or 2000; smaller cities bounced back later or not at all. Average levels of population concentration, as measured by the DDI, were virtually unchanged over the 25-year period from 1990-2015, in contrast to the marked decline in average DDI seen in each decade from 1940 to 1980.

The work reported here has several important limitations. Because the application of Clark's original method to large numbers of urban areas before 1990 is still prohibitively labor intensive, this article has used the data sets of Edmonston (1975) and Edmonston & Guterbock (1984) for its estimates of earlier density functions. Both these datasets are based on painstaking coding of urban shapes to refine two-point estimation methods, but they are less direct and no doubt less accurate than Clark's original method of aggregating small areas into rings (but compare White, 1977). It appears that the DDI estimates produced by these methods are higher than might be attained by the more direct method of Clark, but presumably any bias is consistent across years and therefore will have little effect on estimates of *change* in concentration within each dataset. In addition, the samples examined here are not identical at all points in time, although all are broadly inclusive and the Edmonston/Guterbock sample was designed to give balanced representation to larger and smaller urban areas. It is reassuring to see, in Figure 9, that a constant cohort of cities closely mirrors the pattern of change seen in the full data sets. The geographies used in each of the samples vary: Edmonston's estimates are based on data for Metropolitan Districts (through 1940) and for Urbanized Areas (1950 through 1970); Guterbock and Edmonston's data used here are based on SMSAs 1950 through 1980; while the current estimates ignore official geographies. I have refrained from comparisons at the break points between these samples, and offer these analyses under the assumptions that some errors are mutually self-cancelling when cities are grouped by decade, and that the errors of estimates at the individual city level are not so large as to render the overall longitudinal trends uninterpretable.

Conclusions

The analyses reported here have been confined to a strictly descriptive purpose, seeking to identify changes in urban concentration levels across the years. Substantively, these data show that a master trend of the second half of the twentieth century—suburbanization or the deconcentration of urban areas—started around 1920, accelerated sharply at mid-century and has recently come to an end for the majority of U.S. urban areas. These conclusions bring the study of urban concentration levels in the aggregate into line with common sense and with trends that many have described in smaller-scale studies of urban change, including the recent trend toward urban regeneration (Tallon, 2013).

As the world recovers from the 2020 COVID-19 pandemic, there is a clear possibility that urban form will change once again. It is possible that the demand from households and businesses for central, urban locations may be permanently reduced by the many changes and adaptations the pandemic has caused. As new data on population densities become available, the DDI will be a useful tool in assessing whether or not this worldwide health crisis has triggered a new wave of urban population deconcentration in the United States or in other countries.

Methodologically, these results demonstrate the utility of the DDI as a well-behaved measure of urban deconcentration. Its utility is not confined to the times and places covered here; it will be equally useful for urban areas in other places and other times, that is, for any urban area for which a negative-exponential density-distance function can be estimated with a decent fit.[8] While this article has applied the measure primarily descriptively to evaluate overall trends, the DDI is equally useful for explanatory studies that use urban concentration levels or changes in concentration as the dependent variable, or as a predictor variable. I believe that use of the DDI in future studies could considerably advance the research of those who wish to describe and explain the macro-level differences in urban form in urban areas around the world, and their changes over time.

[8] For example, the weighted negative exponential density function estimates provided for Beijing by Wang and Zhou (1999: table 3) yield DDI scores of 10.48 in 1982 and 10.17 in 1990. Beijing's level of concentration (as measured by DDI) in 1982 was therefore comparable to that of an average U.S. city of 1910, but just eight years later was similar in concentration to a 1940 U.S. city.

References

Barron, J. (2018). New York City's population hits a record 8.6 million. *New York Times,* March 22. Available at: https://www.nytimes.com/2018/03/22/nyregion/new-york-city-population.html (Accessed 12 December 2020).

Berry, B. J. L. & Horton, F.E. (1970). *Geographic perspectives on urban systems.* Englewood Cliffs, NJ: Prentice-Hall.

Bontje, M. (2001). Dealing with deconcentration: Population deconcentration and planning response in poly-nucleated urban regions in north-west Europe. *Urban Studies* 38(4), 769-785.

Broitman, D. & Koomen, E. (2019). The attraction of urban cores: Densification in Dutch city centres. *Urban Studies 57*(9), 1920–1939. https://doi.org/10.1177/0042098019864019

Bromley, R. D., Tallon, A. R. & Thomas, C. J. (2005). City centre regeneration through residential development: Contributing to sustainability. *Urban Studies* 42(13), 2407–2429.

Clark, C. (1951). Urban population densities. *Journal of the Royal Statistical Society* 114(Pt. 4), 490-496.

Duncan, O.D. (1959). Human ecology and population studies. In P. M. Hauser & O. D. Duncan (Eds.). *The study of population* (pp. 678-716). Chicago: University of Chicago Press.

Edmonston, B. (1975). *Population distribution in American cities.* Lexington, MA: Lexington Books.

Edmonston, B. & Guterbock, T. M. (1984). Is suburbanization slowing down? Recent trends in population deconcentration in U.S. metropolitan areas. *Social Forces* 62(4), 905-925.

Frankena, M. W. (1978). A bias in estimating urban population density functions. *Journal of Urban Economics* 5(1), 35-45.

Glickman, N. J. (1979). *The growth and management of the Japanese urban system.* New York: Academic Press.

Guterbock, T. M. (1976). The push hypothesis: Minority presence, crime and urban deconcentration. In Schwartz, B. (Ed.). *The changing face of the suburbs* (pp. 137-161). Chicago, University of Chicago Press.

Guterbock, T. M. (1981). A well-behaved measure of urban deconcentration: The Density Distribution Index. University of Virginia, Metropolitan Deconcentration Project. Working Paper No. 4, Parts I & II. https://uva.theopenscholar.com/tom-guterbock/publications/part-i-well-behaved-measure-population-concentration-density-distribution-index. https://uva.theopenscholar.com/tom-guterbock/publications/part-ii-well-behaved-measure-population-concentration-density-distribution-index. (Accessed 20 July 2021.)

Guterbock, T. M. (1982). Measuring the historical deconcentration of American cities. Presented at the annual meeting of the Social Science History Association, Bloomington IN, November. https://uva.theopenscholar.com/tom-guterbock/publications/measuring-historical-deconcentration-american-cities. (Accessed 20 July 2021.)

Guterbock, T. M. (1990a). The effect of snow on urban density patterns in the United States. *Environment and Behavior* 22(3), 358-386.

Guterbock, T. M. (1990b). Metropolitan population concentration in the United States and Japan. Presented at the fourth annual meeting of the Applied Regional Science Conference, Aoyama Gakuin University, Tokyo, Japan, December 1. https://uva.theopenscholar.com/tom-guterbock/publications/metropolitan-population-deconcentration-united-states-and-japan. (Accessed 20 July 2021).

Hawley, A. H. (1981). *Urban society: An ecological approach.* Second edition. New York: Wiley.

Jackson, K. T. (1975). Urban deconcentration in the nineteenth century: A statistical inquiry. In L. F. Schnore (Ed.). *The new urban history: Quantitative explorations by American historians* (pp. 110-142). Princeton NJ: Princeton University Press.

Juday, L. J. (2015). The changing shape of American cities. University of Virginia, Weldon Cooper Center for Public Service, Demographics Research Group. Available at: https://demographics.coopercenter.org/the-changing-shape-of-american-cities/. (Accessed 20 June 2020).

Kasarda, J. D. & Redfearn, G. (1975). Differential patterns of city and suburban growth in the United States. *Journal of Urban History* 2(1), 43-66. Reprinted in: Berry, B. J. L. & Kasarda, J.D., *Contemporary urban ecology* (chapter 9). New York: Macmillan (1977).

Kawashima, T., Hiraoka, N. & Noro, J. (2014). Recent re-centralization of population in the Tokyo Metropolitan Area: ROXY-index analysis on spatial cycles for the period 1947-2010. In R. Funck & W. Rothengatter (Eds.). *Man, environment, space and time: Economic iInteractions in four dimensions. Karlsruhe Papers in Economic Policy Research* 34, 79-116.

Kotharkar, R. & Bahadure, P. (2020). Achieving compact city form through density distribution: Case of Indian cities. *Journal of Urban Planning and Development* 146(1), 04019022, 1-15.

Kroll, F. & Kabisch, N. (2012). The relation of diverging urban growth processes and demographic change along an urban-rural gradient. *Population, Space and Place* 18(3), 260–276.

Lemoy, R. & Caruso, G. (2020). Evidence for the homothetic scaling of urban forms. *Environment and Planning B: Urban Analytics and City Science* 47(5), 870-888.

Lipton, G. S. (1977) Evidence of central city revival. *Journal of the American Planning Association* 43 (April), 136-147.

McDonald, J. F. (1989). Econometric studies of urban population density: a survey. *Journal of Urban Economics* 26, 361-385

Messick, S. (1989). Validity. In R. L. Linn (Ed.). *Educational measurement, Third Edition* (pp. 13-103). New York: American Council on Education/Macmillan.

Mieszkowski, P. & Mills, E. S. (1993). The causes of suburbanization. *Journal of Economic Perspectives* 7(3), 135-147.

Mills, E. S. (1972a). *Studies in the structure of the urban economy.* Baltimore: Published for Resources for the Future, Inc., by Johns Hopkins University Press.

Mills, E. S. (1972b). *Urban economics.* Glenview IL: Scott, Foresman.

Muth, R. F. (1961). The spatial structure of the housing market. *Paper and Proceedings of the Regional Science Association* 7(1961), 207-20.

Newling, B. (1966). Urban growth and spatial structure: Mathematical models and empirical evidence. *Geographical Review* 56(2), 213-25.

Openshaw, S. (1983). *The modifiable areal unit problem.* Norwick: Geo Books.

Qiang, Y., Xu, J., & Zhang, G. (2020). The shapes of US cities: Revisiting the classic population density functions using crowdsourced geospatial data. *Urban Studies* 57(10), 2147-2162 . DOI: https://doi.org/10.1177%2F0042098019871191.

Rees, P. H. (1968). *The factorial ecology of metropolitan Chicago.* Master's thesis, University of Chicago.

Robson, A. J. & Scheffman, D. T. (1979). Urban structure, gas prices, and the demand for transportation. *Journal of Urban Economics* 6(2), 148-155.

Rogerson, P. A. & Plane, D. A. (2012). The Hoover index of population concentration and the demographic components of change: an article in memory of Andy Isserman. *International Regional Science Review* 1, 97-114.

Rowlands, D. W. (2019). How the D.C. area's population density has changed since 1970. D.C. Policy Center, July 24. Available at: https://www.dcpolicycenter.org/publications/regional-population-density-since-1970/. (Accessed 12 December, 2020).

Schnore, L. M. (1957). The timing of metropolitan deconcentration: A contribution to the debate. Reprinted in L. M. Schnore (Ed.). *The urban scene* (chapter 5). New York: Free Press, 1965.

Tallon, A. (2013). *Urban regeneration in the UK.* London and New York: Routledge.

Thompson, W. S. (1947). *Population: The Growth of Metropolitan Districts in the United States, 1900-1940.* Washington: U. S. Government Printing Office.

Tsai, Y. H. (2005). Quantifying urban form: Compactness or 'sprawl'? *Urban Studies* 42(1), 141-161.

Wang, F. & Zhou, Y. (1999). Modelling urban population densities in Beijing 1982-90: Suburbanization and its causes. *Urban Studies* 36(2), 271-287.

Wang, F., Liu, C. & Xu, Y. (2019). Mitigating the zonal effect in modeling urban population density functions by Monte Carlo simulation. *Environment and Planning B: Urban Analytics and City Science* 46(6), 1061-1078.

White, L. J. (1977). How good are two-point estimates of urban density gradients and central densities? *Journal of Urban Economics* 4(3), 292-309.

[dataset] U.S. Bureau of the Census. 2019. “Annual estimates of the resident population for incorporated places of 50,000 or more, ranked by July 1, 2019 population: April 1, 2010 to July 1, 2019.” Available at: https://www2.census.gov/programs-surveys/popest/tables/2010-2019/cities/totals/SUB-IP-EST2019-ANNRNK.xlsx. (Accessed 12 December 2020).

Zheng, X. P. (1991). Metropolitan spatial structure and its determinants: A case-study of Tokyo. *Urban Studies* 28(1), 87-104.Guter